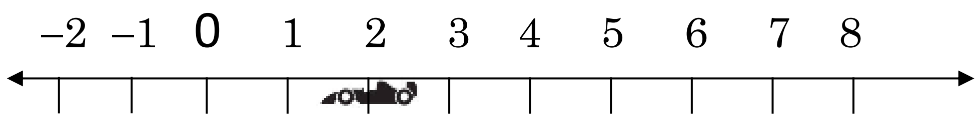
# AVERAGE VELOCITY

Now we look at a number line — a simple horizontal axis — and talk about some object moving left or right on the line.




It could be a racecar speeding down the backstretch or a subatomic proton hurtling through a linear accelerator.

## ❑ NOTATION

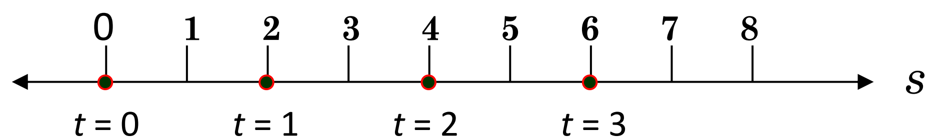
Throughout this chapter, we'll let ***t*** represent ***time*** and ***s*** represent the *position* of the object on the line. We agree that $s > 0$ when the object is to the right of 0, and $s < 0$ on the left side of 0. It is also important that we consider the position, $s$, to be a function of time, $t$. In other words, as the time changes, the position of the object may also change. We can even write the position as $\boldsymbol{s(t)}$ [read: "***s*** of ***t***"] to express the view that the position, ***s,*** is a function of the time, ***t***.

> **Speed v. Velocity**
>
> 65 mph is *speed*.
> 65 mph <u>To The Right</u> is *velocity*.

In addition, to emphasize the math, we won't worry about units. But for your edification, if position is measured in meters from the origin (as either a positive number, negative number, or zero) and time is measured in seconds (common in physics), then the unit of **velocity** is *meters per second*, or m/s, or $\frac{\text{m}}{\text{s}}$, followed by the direction "left" or "right."

## ❑ TIME AND POSITION

Consider the following real line (which we could call the $s$-axis). Each dot on the line represents the **position** of an object at the moment of **time** specified, where $t$ might be measured in seconds:




Here's what this little picture means. At the "beginning," when $t = 0$, the position is $s = 0$. One second later, when $t = 1$, the object is at $s = 2$. One second after that, when $t = 2$, $s$ has the value 4. When $t = 3$, the object is at 6. Expressing these facts in function notation, we write:

$s(0) = 0$ $\quad s(1) = 2$ $\quad s(2) = 4$ $\quad s(3) = 6$

[The notation ***s*(*t*)** is read "*s* of *t*."]

We can also express this number line picture with a table of values:

| $t$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $s$ | 0 | 2 | 4 | 6 |

Our goal now is to predict where the object will be at some time in the **future.** For example, when $t = 10$, that is, 10 seconds after the object begins moving, where will it be? To do this, we ask the question: Given the data in the table, can we find a formula for the position of the object as a function of time? Looking either at the real line (the $s$-axis) or the table, it seems (at least from the data we have) that the position is always twice the value of time: ***s* is twice *t***. So we can write the formula

$s(t) = 2t$ [The position at time $t$ is twice the $t$-value]

Thus, to predict the position $s$ when $t = 10$, all we have to do is calculate $s(\mathbf{10})$, which is $2 \cdot \mathbf{10} = 20$. Therefore, after 10 seconds have passed, the object is at **position 20**. Notice that the object in this example never has a negative value of $s$, and that it moves to the right as the value of time increases.

## Homework

1. The following table represents the position of a car at various moments in time. Predict the position of the car when $t = 850$.

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $s$ | 4 | 7 | 12 | 19 | 28 |

## ❑ *AVERAGE VELOCITY*

Have you studied **delta** notation? A change in $y$, which might be written as the difference $y_2 - y_1$, can be expressed as $\Delta y$. In discussing velocity, we need the notion of a change in time, denoted by $\Delta t$, and as a consequence, a change in position, written $\Delta s$.

The symbol **Δ** is the Greek capital letter delta, and is a common symbol representing a **change** in something.

Now we're ready to define the concept of ***average velocity***:

**average velocity = the change in position divided by the change in time**.

It's a measure of the *net distance* traveled in a specified amount of time. We use the symbol $\overline{v}$ ($v$-bar) to represent average velocity:

$$\overline{v} = \frac{\Delta s}{\Delta t}$$

This formula is very similar to the formula involving distance, rate, and time:

$$r = \frac{d}{t}$$

**Four Notes:**

1. The change in position, $\Delta s$, does <u>not</u> take into account the actual distance traveled by the object. Suppose you and I are standing on the $s$-axis at the same spot, say $s = 10$. You run many miles to the left and then turn around and run back, ending at $s = 12$. Your change in position is $12 - 10 = \mathbf{2}$. I, being the lazy bum that I am, simply stroll directly from $s = 10$ to $s = 12$. My change in position is $12 - 10 = \mathbf{2}$, the same change in position as yours. We started at the same spot and we ended at the same spot, and so we each experienced the same change in position — our $\Delta s$'s are the same (even though you ran a much farther distance than I did.)

2. The formula for average velocity may remind you of the definition of **slope**, $\frac{\Delta y}{\Delta x}$. This is no coincidence. Each of the concepts — *slope* and *average velocity* — is described by a ratio of changes, and each of them forms the foundation for the branch of math called *calculus*.

3. Just as the slope of a line can be negative, so too can the average velocity be negative. You'll see this in Example 2 below. By the way, this is one of the reasons that we refer to the theme of this chapter using the term *velocity* rather than speed.

4. You might notice at this point that your calculation for average velocity will involve two subtractions followed by a division. The order in which you perform the subtractions is up to you, but you MUST be consistent in that order. Nevertheless, tradition dictates that we subtract initial values from final values. For example, if an object is at $s = 3$ and then later is at $s = 10$, we agree to calculate $10 - 3$ rather than $3 - 10$. So our average velocity formula looks like

Velocity has direction; speed does not.

$$\bar{v} = \frac{\textit{final position} - \textit{initial position}}{\textit{final time} - \textit{initial time}}$$

<u>EXAMPLE 1</u>: **A bird takes off from position $s = 3$ when $t = 5$ and lands at position $s = 15$ when $t = 9$. Find the average velocity for the time interval $t = 5$ to 9.**

<u>Solution</u>: The average velocity is the change in position (the difference of the positions) divided by the change in time (the difference of the times):

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{15-3}{9-5} = \frac{12}{4} = \boxed{3}$$

<u>EXAMPLE 2</u>: **A racecar is at position $s = 8$ when $t = 0$ and then drives to position $s = -10$ in 2 seconds. Find the average velocity during the 2-second interval.**

<u>Solution</u>: The racecar started when $t = 0$ and ended when $t = 2$. Using the formula for average velocity, we calculate

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{-10-8}{2-0} = \frac{-18}{2} = \boxed{-9}$$

Why is the velocity negative?

# Homework

2. A snail is at position $s = 22$ at $t = 4$ and moves to position $s = 50$ at $t = 18$. Find the average velocity, $\bar{v}$, for the time interval $t = 4$ to 18.

3. A rocket is at position $s = 1{,}000$ at $t = 20$ and rises to position $s = 5{,}500$ at $t = 65$. Find the average velocity, $\bar{v}$.

EXAMPLE 3: **Let $s(t) = t^2$. Calculate each of the following:**

**a. $s(0)$ b. $s(20)$ c. $\bar{v}$ from $t = 5$ to $t = 15$**

Solution:

a. $s(0)$ means let $t = 0$ and calculate $s$ using the given formula:

$s(0) = 0^2 = \mathbf{0}$

b. s(20) is the position when $t = 20$:

$s(20) = 20^2 = \mathbf{400}$

| $t$ | $s$ |
|---|---|
| 0 | 0 |
| 5 | 25 |
| 15 | 225 |
| 20 | 400 |

c. i) $s(5) = 5^2 = 25$

ii) $s(15) = 15^2 = 225$

The average velocity, $\bar{v}$, is calculated as follows:

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{225 - 25}{15 - 5} = \frac{200}{10} = \mathbf{20}$$

# Homework

For each position function, calculate the average velocity, $\bar{v}$, during the given time interval:

4. $s(t) = 3t + 7$ $t = 3$ to $t = 10$

5. $s(t) = t^2 + 3t + 7$ $t = 1$ to $t = 5$

6. $s(t) = t^3$ $t = 3$ to $t = 5$

7. $s(t) = t^3 + t^2 + 3t + 7$ $t = 1$ to $t = 2$

8. $s(t) = \frac{1}{t}$ $t = 4$ to $t = 6$

9. $s(t) = \sqrt{t}$ $t = 9$ to $t = 36$

10. $s(t) = \frac{1}{\sqrt{t}}$ $t = 4$ to $t = 100$

11. $s(t) = 7$ $t = \pi$ to $t = 50$

12. For each position function, calculate the average velocity, $\bar{v}$, during the time interval from $t = 0$ to $t = 1$:

a. $s(t) = at + b$

b. $s(t) = at^2 + bt + c$

c. $s(t) = at^3 + bt^2 + ct + d$

# Solutions

**1.** 722,503

**2.** 2

**3.** 100

**4.** 3

**5.** 9

**6.** 49

**7.** 13

**8.** –0.04

**9.** 0.11

**10.** –0.004

**11.** 0

**12.** I want to see what you come up with.

**“STRIVE FOR** *progress,*
**NOT** *perfection.”*